Area Related to Circle

Free Response Questions

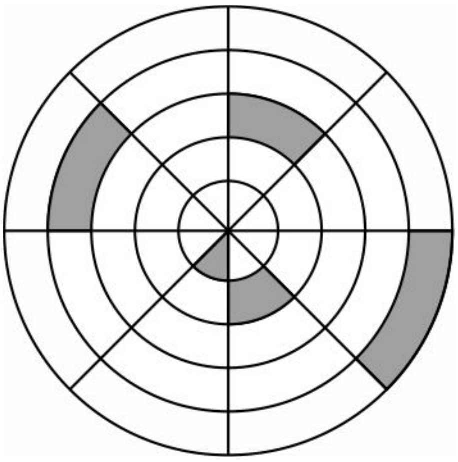
Q: 1 **Rohan got the question below in his school test.** **[1]**

"A 7 cm chord of a circle subtends an angle of 60° at the centre. Find the area of the major sector."

After the test, he went to the teacher and said, "The question cannot be answered as it doesn't mention the radius of the circle".

Is Rohan right or wrong? Give a valid reason for your answer.

Q: 2 **Shown below is a figure such that each circle is divided into equal sectors by 4 straight lines. The combined area of the shaded region is 77 square units.** **[1]**



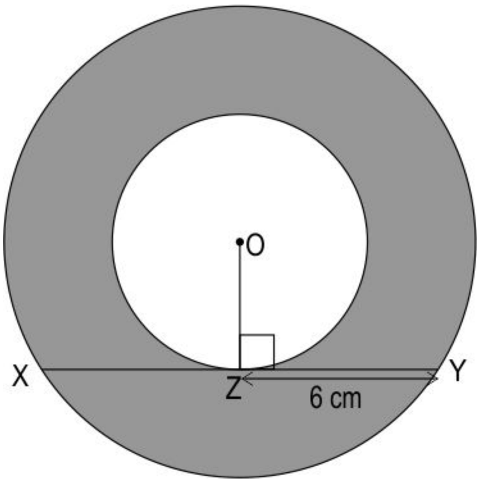
What is the radius of the largest circle? Show your work.

(Note: Take pi as $\frac{22}{7}$.)

Q: 3 **A chord of length 2√2 cm subtends an angle of 90° at the centre of a circle.** **[1]**

Find the area of the minor sector in terms of π. Show your work.

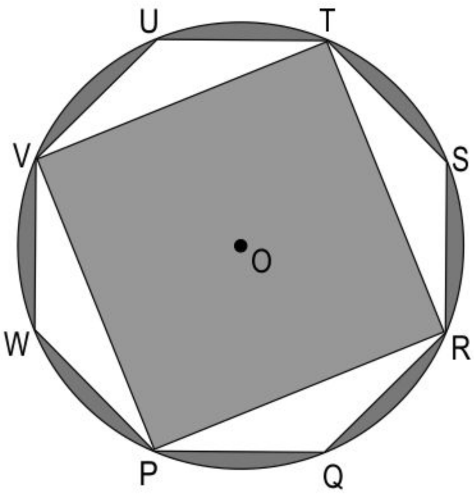
Q: 4 **Shown below are two concentric circles with centre O. XY is tangent to the inner circle at Z.** **[1]**



(Note: The figure is not to scale.)

What is the area of the shaded region in terms of π? Show your work.

Q: 5 **A regular octagon of side length 4 cm is inscribed in a circle of radius 7 cm. A square is inscribed in the same circle as shown below.** **[5]**

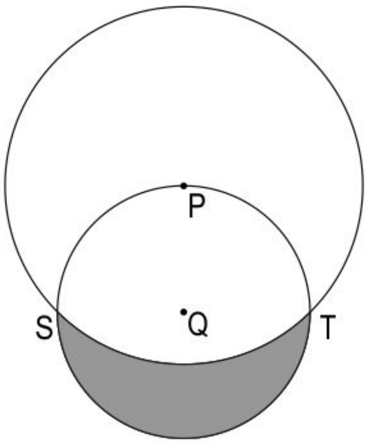


(Note: The figure is not to scale.)

Find the area of the shaded region. Show your work.

(Note: If needed, take pi as $\frac{22}{7}$, √3 as 1.7, √5 as 2.2.)

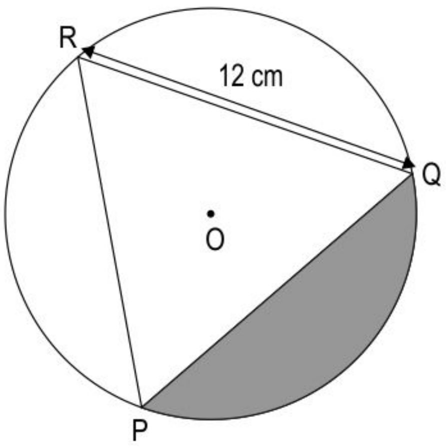
Q: 6 **Shown below are two circles with centres P and Q. Diameter ST is 6 cm.** **[5]**



(Note: The figure is not to scale.)

Find the area of the shaded region. Draw a rough diagram and show your work.

Q: 7 **Shown below is a circle with centre O. PQR is an equilateral triangle of side length 12 cm.** **[5]**



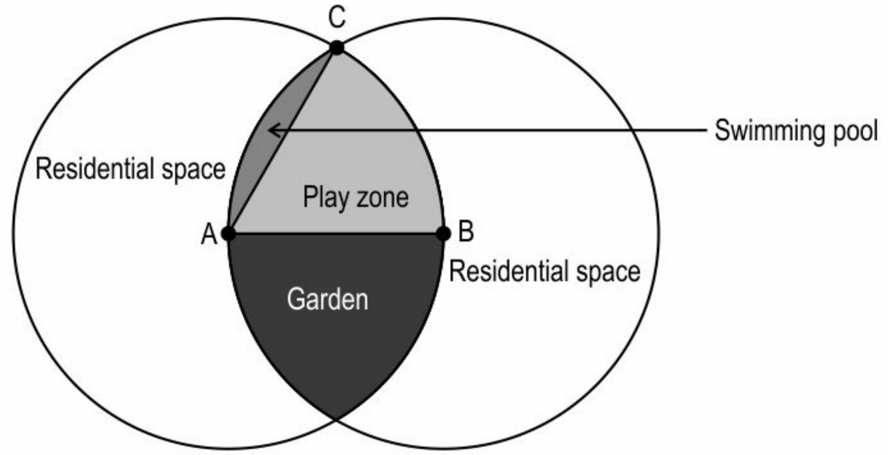
(Note: The figure is not to scale.)

Find the area of the shaded region in terms of π. Draw a rough diagram and show your work.

Case Study

Answer the following questions using the given information.

Shown below is a representation of some portion of an apartment complex. The area occupied by this portion is in the form of two identical intersecting circles whose centres are at A and B respectively. The radius of each circle is 21 metres. The intersecting area is converted into a recreational space comprising of a play zone, a garden and a swimming pool.



(Note: Take π as $\frac{22}{7}$).

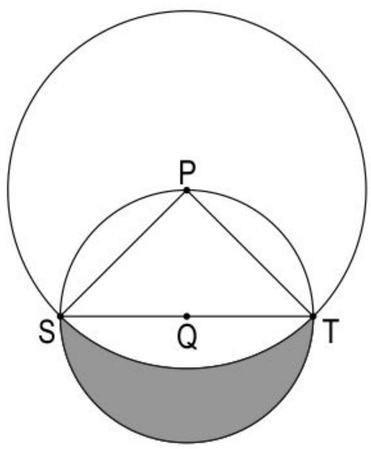
Q: 8 **Find the area of the play zone. Show your work.** **[2]**

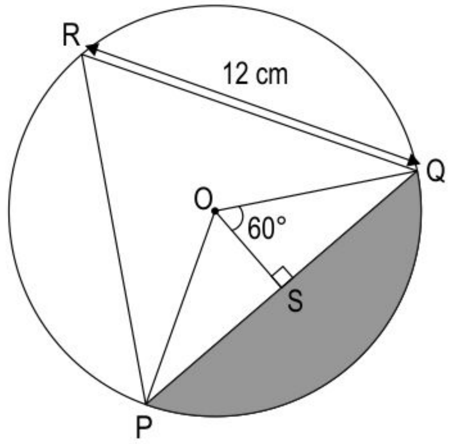
Q: 9 **The boundary of the recreational space is installed with some landscape lighting that costs is Rs 2500 per metre.** **[2]**

How much did it cost to install the lighting along the boundary? Show your work with valid reasons.

Q: 10 **If the area of the recreational space is 842 m^2, find the area of the residential space in the apartment. Show your work.** **[1]**

Q.No	What to look for	Marks
1	**Writes that Rohan is wrong and gives a reason. For example, since the chord extends an angle of 60° at the centre, it forms an equilateral triangle with the radii and hence, the radius is 7 cm.**	**1**
2	**Identifies that the shaded regions combined forms a sector, assumes the radius of the largest circle as *r* units and writes the equation as:** $\frac{1}{8} \times \frac{22}{7} \times r^2 = 77$	**0.5**
	Solves the above equation to find the value of *r* as 14 units.	**0.5**
3	**Identifies that the radii (*r* cm) and the chord make an isosceles right triangle, and uses the Pythagoras theorem to write:** $r^2 = 4\ cm^2$	**0.5**
	Finds the area of the minor sector as: $\frac{90}{360} \times \pi \times 4 = \pi\ cm^2$	**0.5**
4	**Assumes the radii of the outer and inner circles as R cm and *r* cm respectively. Writes the expression for the area of the shaded region as:** $\pi(R^2 - r^2)\ cm^2$	**0.5**
	Uses the Pythagoras theorem to find $(R^2 - r^2)\ cm^2$ **as 36** cm^2 **and hence finds the required area as** $36\pi\ cm^2$**.**	**0.5**
5	**Writes that the octagon divides the circles into 8 equal sectors of 45° each and finds the area of each of the sectors as:** $\frac{45}{360} \times \frac{22}{7} \times 7 \times 7 = \frac{77}{4}$ **or 19.25** cm^2**.**	**1**
	Finds the semiperimeter of ΔVOU as $\frac{7+7+4}{2} = 9$ **cm.**	**0.5**
	Finds the area of ΔVOU as $\sqrt{(9 \times 2 \times 2 \times 5)} = 6\sqrt{5} = 13.2\ cm^2$**.**	**0.5**
	Finds the area of the 8 segments as $8(19.25 - 13.2) = 48.4\ cm^2$**.**	**1**

Q.No	What to look for	Marks
	Finds the area of square PRTV as $\frac{14}{\sqrt{2}} \times \frac{14}{\sqrt{2}} = 98$ cm^2.	**1**
	Finds the area of the shaded region as $48.4 + 98 = 146.4$ cm^2. **(Award full marks if an alternative approach is used to answer correctly.)**	**1**
6	**Draws a rough diagram. The figure may look as follows:**  ***(Note: The figure is not to scale.)***	**1**
	Uses properties of angles in a semicircle and writes that $\angle SPT = 90°$.	**0.5**
	Uses pythagoras theorem in ΔSPT where SP = PT to write: **$SP^2 + PT^2 = 6^2$** **=> $SP^2 = 18$ cm^2**	**1**
	Finds the area of ΔSPT as $\frac{1}{2} \times 18 = 9$ cm^2.	**0.5**
	Finds the area of sector PST in circle with centre P as $\frac{90}{360} \times \pi \times 18 = \frac{9\pi}{2}$ cm^2.	**1**
	Finds the area of semicircle with diameter ST as $\frac{1}{2} \times \pi \times 3^2 = \frac{9\pi}{2}$ cm^2.	**0.5**
	Finds the area of the shaded region as $\frac{9\pi}{2} - \frac{9\pi}{2} + 9 = 9$ cm^2.	**0.5**

Q.No	What to look for	Marks
7	**Draws a rough figure. The figure may look as follows:**  ***(Note: The figure is not to scale.)***	**1**
	Writes that angle subtended by a chord at the centre is double that of the circumference and hence finds $\angle POQ$ as 120°.	**0.5**
	Writes that OS is perpendicular to PQ, hence $\angle QOS = 60°$ and SQ = 6 cm.	**0.5**
	Finds the length of the radius, OQ as $\frac{6}{\sin}$ 60°)} = $4\sqrt{3}$ cm.	**0.5**
	Finds the length of OS as $4\sqrt{3} \times \cos 60° = 2\sqrt{3}$ cm. **(Award full marks if pythagoras theorem is used correctly.)**	**0.5**
	Finds the area of ΔOPQ as $\frac{1}{2} \times 12 \times 2\sqrt{3} = 12\sqrt{3}$ cm^2.	**0.5**
	Finds the area of sector POQ as $\frac{120}{360} \times \pi \times 4\sqrt{3} \times 4\sqrt{3} = 16\pi$ cm^2.	**1**
	Finds the area of the shaded region as $(16\pi - 12\sqrt{3})$ cm^2.	**0.5**
8	**Writes that AB, BC and CA being radii of identical circles, are equal making ΔABC an equilateral triangle.** **Hence concludes that $\angle CAB = 60°$.**	**1**

Q.No	What to look for	Marks
	Finds the area of the sector forming the playzone as: $\frac{60}{360} \times \frac{22}{7} \times 21^2 = 231 \text{ m}^2$	**1**
9	**Writes that AB, BC and CA being radii of identical circles are equal making ΔABC an equilateral triangle.** **Hence concludes that $\angle BAC = 60^\circ$.**	**0.5**
	Finds the arc length BC as: $\frac{60}{360} \times 2\pi \times 21 = 22 \text{ m}$	**0.5**
	Calculates the length of the boundary or perimeter of the recreational space as: $4 \times$ **length of arc BC** $= 4 \times 22 = 88$ **m**	**0.5**
	Finds the cost of installing the landscape lighting as 88 x 2500 = Rs 220000.	**0.5**
10	**Finds the area of each circle as** $\frac{22}{7} \times 21^2 = 1386 \text{ m}^2$.	**0.5**
	Finds the area of the residential space as: **($2 \times$ area of circle) - ($2 \times$ area of recreational space)** $= 2 \times (1386 - 842)$ $= 1088 \text{ m}^2$	**0.5**